MILLIKAN OIL-DROP EXPERIMENT

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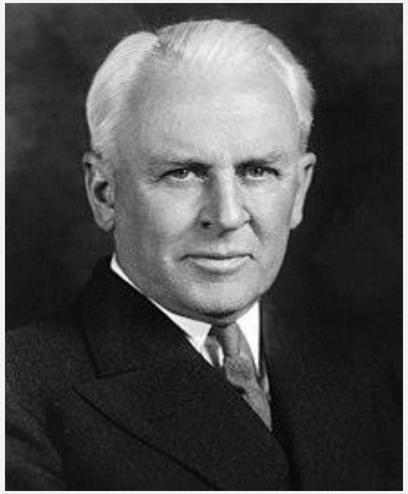
Classe: 5 sez. D

Liceo Scientifico G. Peano

HISTORICAL INTRODUCTION

- The oil-drop experiment was performed by Robert Andrews Millikan (1868-1953) and his then-graduate student Harvey Fletcher (1884-1981) to measure the charge of the electron.
- It was performed in the Ryerson Physical Laboratory at the University of Chicago.
- The existence of the electron was getting widely accepted:
    - **Wilhelm Röntgen** (1845-1923)→X-rays
    - **Henri Becquerel** (1852-1908)→radioactivity
    - **J.J. Thomson** (1856-1940)→cathode rays were streams of particles→e/m



HISTORICAL INTRODUCTION

- Between **1904** and **1910** Millikan performed several experiments: he analysed the motion of water droplets with the student Louis Begeman.
- Millikan obtained more accurate results with the oil-drop experiment in **1910**: **Harvey Fletcher** suggested to replace water with oil.
- They published their final results in **1913**.
- Millikan was honoured with the Nobel Prize for Physics in **1923** for his measurement of the elementary electronic charge and for his contribution on the photoelectric effect.

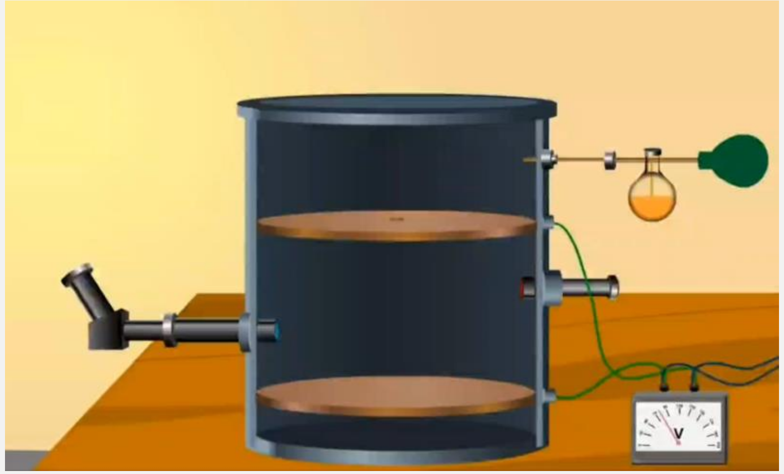
HISTORICAL INTRODUCTION

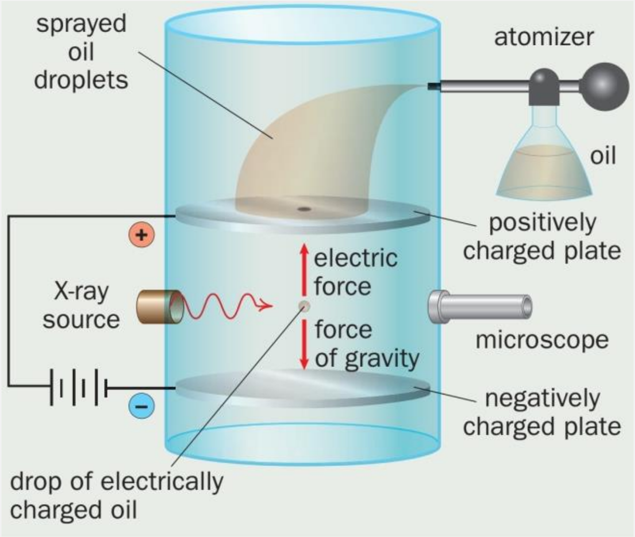
Millikan said in his Nobel Lecture about his work:

“My own work has been that of the mere experimentalist whose main motive has been to devise, if possible, certain crucial experiments for testing the validity or invalidity of conceptions advanced by other regarding the unitary nature of electricity”

APPARATUS USED BY R.A. MILLIKAN

- The experimental apparatus was made up of a **containter** with a **capacitor** inside.
- Oill droplets are sprayed in the capacitor through a small perforation by an **atomizer**.
- The motion of the oil drops can be observed through a **microscope with a graduated scale**.





APPARATUS USED BY R.A. MILLIKAN

- The apparatus includes a **light source** which illuminates the oil drops making them appear as bright stars.
- Quoting Millikan's and Fletcher's own words:

"I saw a most beautiful sight. The field was full of little starlets, having all colors of the rainbow. The larger drops soon fell to the bottom, but the smaller ones seemed hang in the air for nearly a minute. They executed the most fascinating dance"

(H. Fletcher)

"The appearance of this drop is that of a brilliant star on a black background."

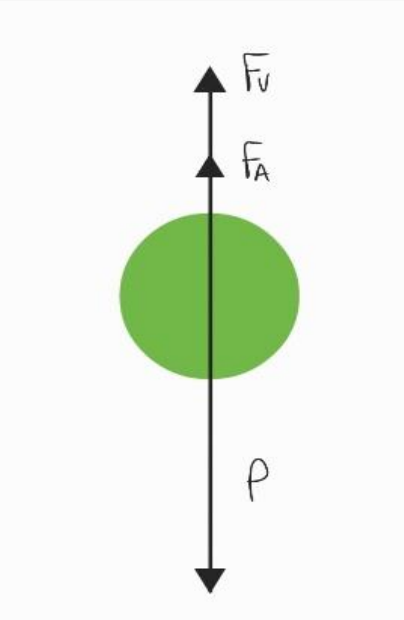
(R.A. Millikan)

THE EXPERIMENT – FIRST PHASE

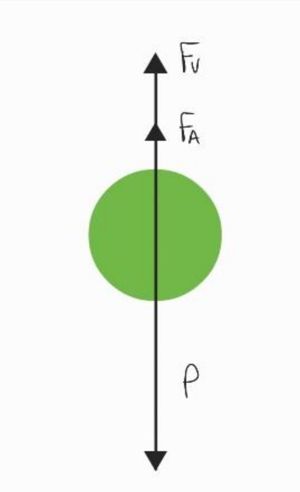
Millikan experiment can be divided into three phases.

- In the **first phase** there is no electric field applied and Milllikan could calculate the radius of the drops.
- Three forces act on each droplet:

1. Gravitational force (P)
2. Archimede's force (F_A)
3. Friction with air (F_V)



THE EXPERIMENT – FIRST PHASE



- The equation of motion:

$ma = P - F_A - F_v$, where $F_v = 6\pi\eta r v$ (Stokes' law)

$$ma = mg - V\varrho_{air}g - 6\pi\eta r v = V\varrho_{oil}g - V\varrho_{air}g - 6\pi\eta r v$$

$$= Vg\,(\varrho_{oil} - \varrho_{air}) - 6\pi\eta r v = V\varrho g - 6\pi\eta r v$$

$$\rightarrow \boldsymbol{V\varrho g = 6\pi\eta r v_1} \rightarrow \boldsymbol{v_1} = \frac{V\varrho g}{6\pi\eta r} = \frac{4}{3}\pi r^3 \frac{\varrho g}{6\pi\eta r} = \frac{2}{9}\frac{\varrho g r^2}{\eta}$$

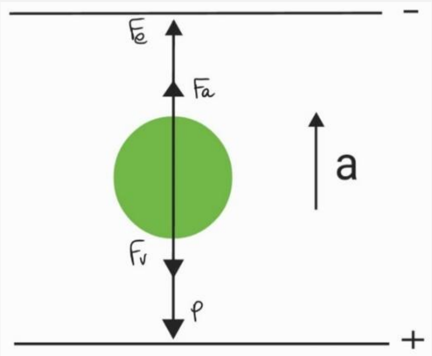
THE EXPERIMENT – SECOND PHASE

- In the **second phase** Millikan applied an electric field→there was a fourth electric force acting on each on the oil drops.
- Millikan could observe several motions of the droplets and so he could achieve a lot of data.
- Fletcher described what he saw after turning on the electric field for the first time in the oil-drop experiment:

"As I looked through the telescope I could see the tiny stream of oil droplets coming through the hole… As soon as I turned on the switch some of them went slowly up and some went faster down. I was about to scream as I knew then some were charged negatively and others positively. By switching the field off and on with the right timing one could keep a selected droplet in the field of view for a long time." (H. Fletcher, unpublished autobiography)

THE EXPERIMENT – SECOND PHASE

- We can focus the attention on a drop charged positively with a strong electric potential force that can overcome the other forces. The droplet is pulled upwards because of the electric force.



- The equation of motion is:

$$ma = F_e + F_A - F_V - P$$

$$\rightarrow \boldsymbol{qE - \varrho V g = 6\pi\eta r v_2}$$

$$\boldsymbol{v_2} = \frac{qE - \varrho V g}{6\pi\eta r} = \frac{qE}{6\pi\eta r} - v_1$$

It is possible to calculate q from this from this expression but Millikan wanted to prove the quantization of electronic charge

THE EXPERIMENT – SECOND PHASE

- Millikan had to find the **greatest common divisor** for the many values of charges he obtained.

↓

- He placed X-rays into air in the capacitor in order to produce an ionization of particles and modify the charge of the oil drops, since the charge on a single electron is minimal.

THE EXPERIMENT – THIRD PHASE/ANALYSIS OF RESULTS

- In the **third phase**, we can measure the difference Δq.

$$\boldsymbol{v_3} = \frac{q'E - \varrho V g}{6\pi\eta r} = \frac{q'E}{6\pi\eta r} - v_1 \rightarrow v_3 - v_2 = \frac{\Delta q E}{6\pi\eta r}$$

- Millikan and Fletcher repeated the experiment many time and they determined the charge of the electron to be 4.774(5) (±0.005) x 10^{-10} statcoulombs (**1.5924 x 10^{-19} coulombs**). These values were really accurate: their calculations are within 1% of todays's current accepted value of the electron's charge, which is **1.60217653(14) x 10^{-19} coulombs.**

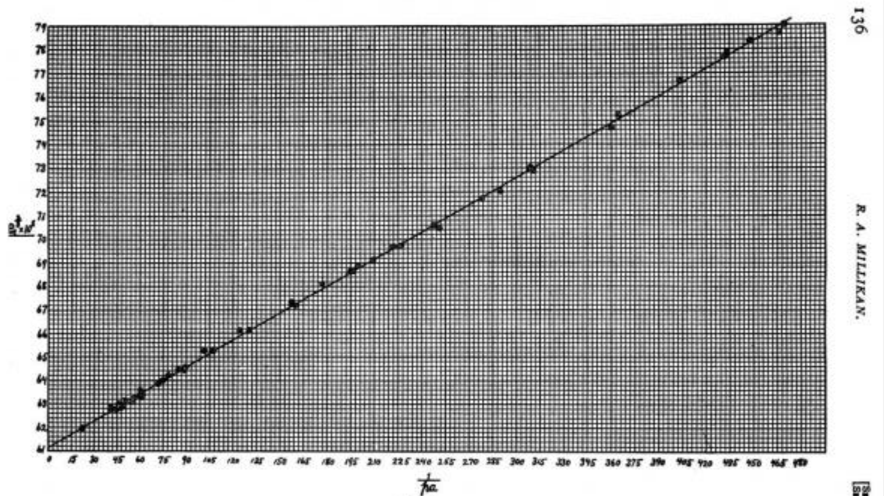
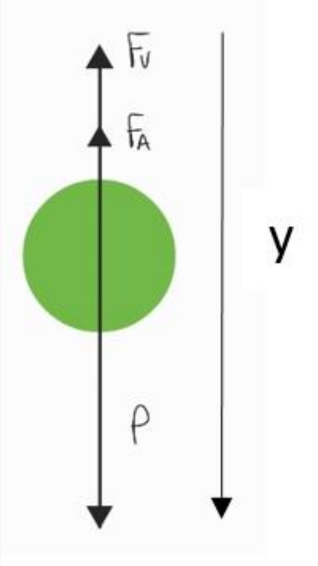


Fig. 2.

ANALISI DEL MOTO DI UNA PARTICELLA CHE CADE ATTRAVERSO UN MEZZO VISCOSO



- Consideriamo la prima fase dell'esperimento di Millikan. Per analizzare il moto di caduta della goccia, con gli strumenti dell'analisi matematica, prendiamo la direzione positiva dell'asse y rivolta verso il basso.
- L'equazione del moto è la seguente:

$$ma = P - F_A - F_v$$

$$ma = mg - V\varrho_{aria}g - 6\pi\eta r v \rightarrow ma = V\varrho_{olio}g - V\varrho_{aria}g - 6\pi\eta r v = Vg\,(\varrho_{olio} - \varrho_{aria}) - 6\pi\eta r v = V\varrho g - 6\pi\eta r v$$

⇩

$$\frac{d^2y}{dt^2} + \frac{6\pi\eta r}{m}\frac{dy}{dt} - \frac{V\varrho g}{m} = 0 \rightarrow \frac{d^2y}{dt^2} + A\frac{dy}{dt} + B = 0 \quad (1)$$

ANALISI DEL MOTO DI UNA PARTICELLA CHE CADE ATTRAVERSO UN MEZZO VISCOSO

- Si calcolano a parte le soluzioni della medesima equazione associata omogenea ($r^2 + Ar = 0$); esse sono $r_1 = 0$ e $r_2 = -A$.

- La funzione spostamento, $y(t)$, derivata da questo calcolo differenziale, sarà del tipo:

$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t} + g(t) = c_1 e^{0t} + c_2 e^{-At} + g(t) = c_1 + c_2 e^{-At} + g(t)$$

- $g(t) = kt$ → si ha che: $g'(t) = k$ e $g''(t) = 0$. Sostituendo in (1) si ottiene che: $k = -\frac{B}{A}$.

⇩

$$y(t) = c_1 + c_2 e^{-At} - \frac{B}{A} t \rightarrow y'(t) = -Ac_2 e^{-At} - \frac{B}{A}$$

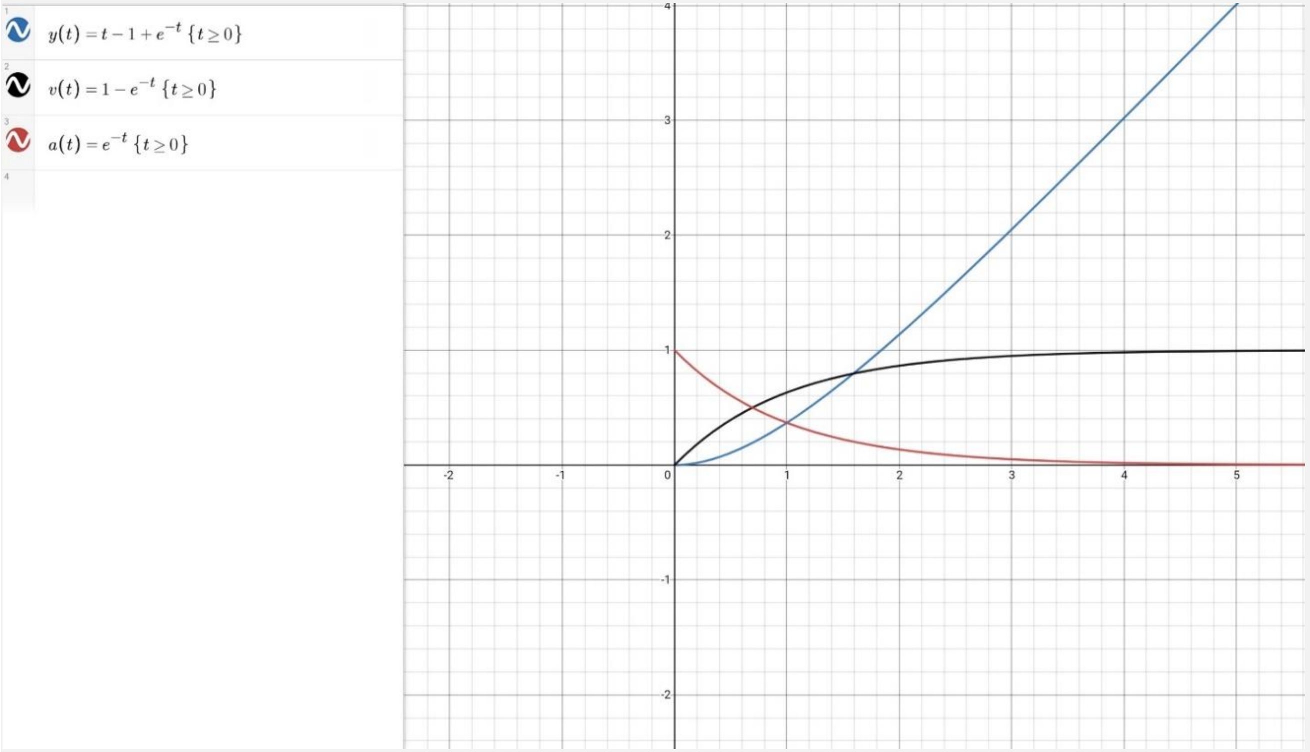
- Per determinare c_1 e c_2 si risolve il seguente sistema tenendo conto delle condizioni iniziali:

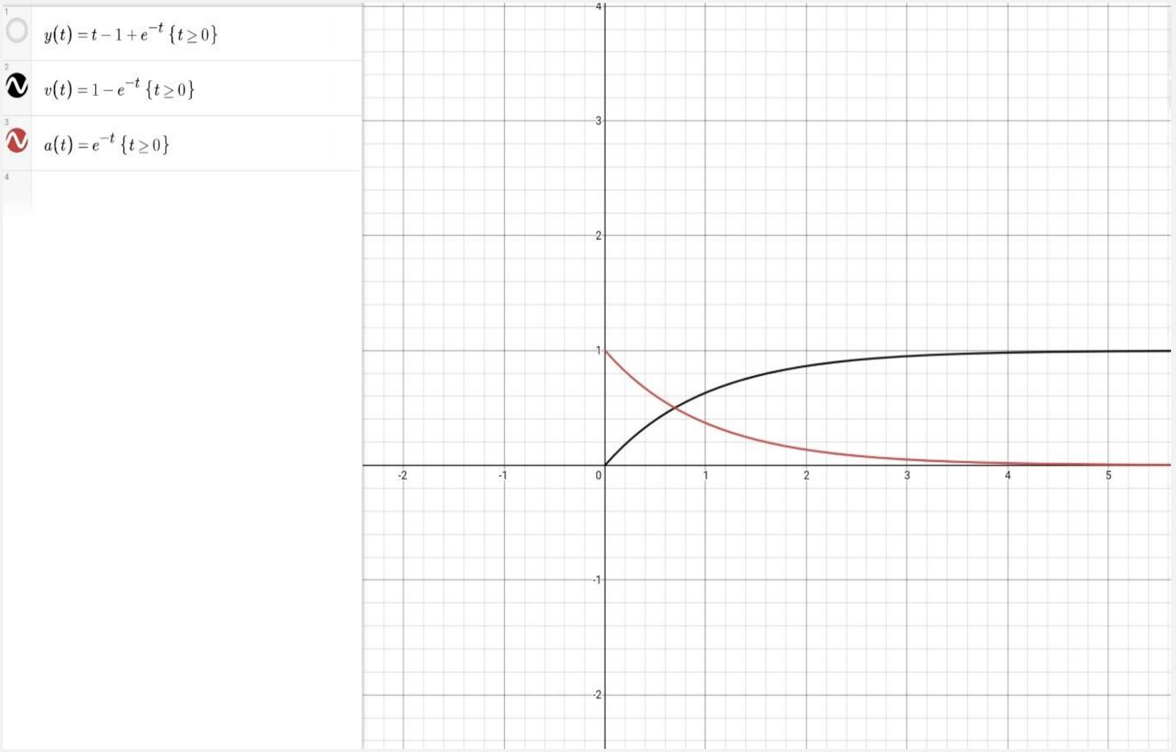
$$\begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases} \begin{cases} c_1 + c_2 = 0 \\ -Ac_2 - \frac{B}{A} = 0 \end{cases} \begin{cases} c_1 = \frac{B}{A^2} \\ c_2 = -\frac{B}{A^2} \end{cases}$$

ANALISI DEL MOTO DI UNA PARTICELLA CHE CADE ATTRAVERSO UN MEZZO VISCOSO

- Tenendo conto di questi valori trovati e ponendo A = 1 e B = -1 si ottiene:

$y(t) = t - 1 + e^{-t}$, derivando rispetto al tempo: $v(t) = 1 - e^{-t} \;\rightarrow\; a(t) = e^{-t}$





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