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a.s. 2020-2021

Candidato: CANONICO Martina   Classe: 5 sez. D

Discipline di indirizzo: MATEMATICA E FISICA

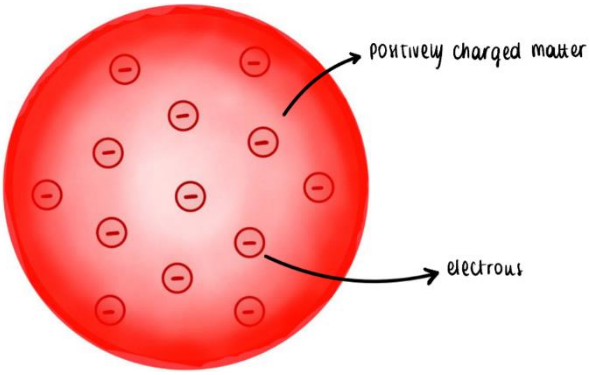
ARGOMENTO
RUTHERFORD EXPERIMENT

The candidate has to provide a historical introduction to the topic, describe the apparatus used by Geiger and Marsden and the experimental results.
The student should explain the reasons why these results were so astonishing and the deductions that Rutherford drew from the experimental data.
The candidate has to represent the function that describes the distribution of the alpha particles and analyse its physical meaning.

Commissari: Proff. Laura di Siena, Fabrizia de Bernardi

Historical introduction

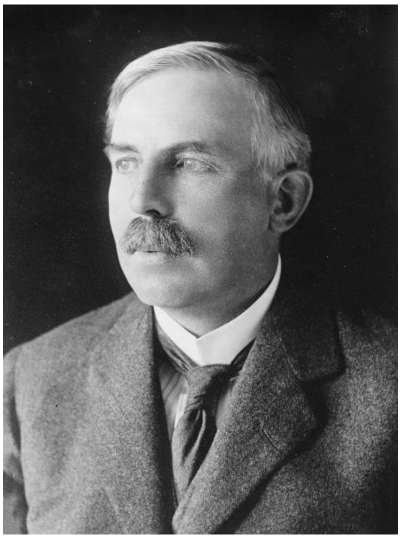
By the beginning of the 20th century, the Physics community had launched research about atomic structure. Thomson's experiment (1897) gave an important boost to this scientific research. The most important discoveries bestowed to him are the discovery of the first elementary particle (later named as "electron"), the quantization of the electric charge and the existence of particles smaller than atoms. As consequence of his work, he theorized an atomic model, known as "plum pudding model".



According to this model (1900s), the atom can be thought as a sphere with a uniformly distributed positive charge (the pudding) in which there are electrons (the plumbs) that neutralize the positive charge.

Rutherford and his research partners Geiger and Marsden (an undergraduate), investigating the α-particle scattering by a gold foil, found astonishing results that led to a new idea of the atomic structure.

Short biography



Ernest Rutherford was born in New Zealand on 30th august 1871. His parents were farmers immigrated from Scotland. At school, he demonstrated to have a flair for Maths and Physics and thanks to his brilliant attitude he could come to Europe to attend the University of Cambridge, where he was introduced by J.J. Thomson to Cavendish Laboratory. It is said that Rutherford was used to give his parents a hand in the fields and when he was told that he had won a scholar ship he watched a potato and he said "This is the last potato I pick up". At university, he examined in depth Becquerel's results about radioactivity and X-rays effects on electrical conduction through gases. In 1908 he won the Nobel Prize in Chemistry because of his investigation about the chemistry of radioactive substances. He worked with cathode rays tube and proved that X-rays and radioactivity can ionise gases or air. In normal conditions, air is not a good conductor, unless when it is ionized. In this case, electric charges are generated and so gas carries an electric current. He distinguished two kinds of radioactivity:

1. Alpha rays: alpha particles are positive helium ions. Nowadays, it is known that α-particles are helium nuclei with two protons and two neutrons. Their charge is equal to 2e (where $e = 1{,}60 \cdot 10^{-19}\ C$). α-particles are due to the decay of Radium.
   Rutherford, Geiger and Marsden didn't know how tiny these nuclei were and, above all, they didn't either know the existence of nucleus.
2. Beta rays: ß-particles can be electrons or positrons (electrons with a positive charge) depending on the type of decay.

Apparatus of the experiment

The three scientists (especially Geiger and Marsden, because Rutherford wasn't expecting much from this experiment) directed their research program to study the scattering of α-particles passing through thin foils of gold.

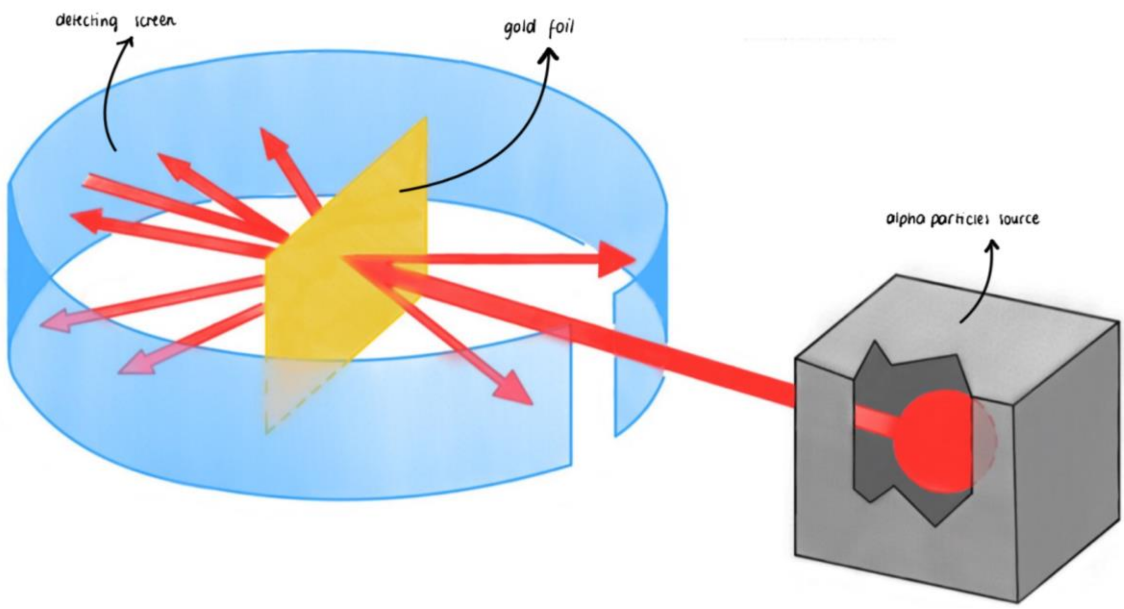
In this experiment α-particles are fired at a thin sheet of gold foil at different angles to analyse the structural properties of the atom.

Rutherford expected α-particles fired at the atoms of gold to be only slightly deflected. What in fact happened was that most of them passed straight through without changing direction, and only a few (about 1 in 8000-10000) were deflected (or "scattered") at different angles. Some of these deflected alpha particles came bouncing back in the opposite direction, as though they had hit something dense and solid in the gold foil. This was such a shock to Rutherford that he said:

> *"It was almost as you fired a fifteen-inch shell at a piece of tissue paper and it bounced back and hit you"*

If Thomson's idea of the atom were correct, the positive mass in the gold foil would be relatively diffuse and would allow the alpha particles to pass through the foil with no scattering or, at least, only minor scattering. Rutherford arranged to calculate the trajectories of the scattered particles. First, he had to verify if some alpha particles could be deflected at angles greater than 90°. This fact was very unlikely because these particles have a big mass and carry a great energy (about 5 Mev).

He formulated his orbital atomic model from these observations, and he concluded that the "plum pudding" model was incorrect.



The experimental apparatus consisted in:

- Alpha particles source
  α-particles are produced by the radioactive decay of a piece of Radium (^{241}Am, isotope of Americium, it is radioactive). It is closed in a lead shielding, that has a small slit, to obtain a collimated beam of particles.
- Vacuum chamber
- Target gold foils
- Detecting screen
  It is a circular fluorescent screen, covered by zinc sulphide (ZnS). The particles issue a flash of light when they hit the screen.

The pump is turned on to obtain vacuum in the chamber.

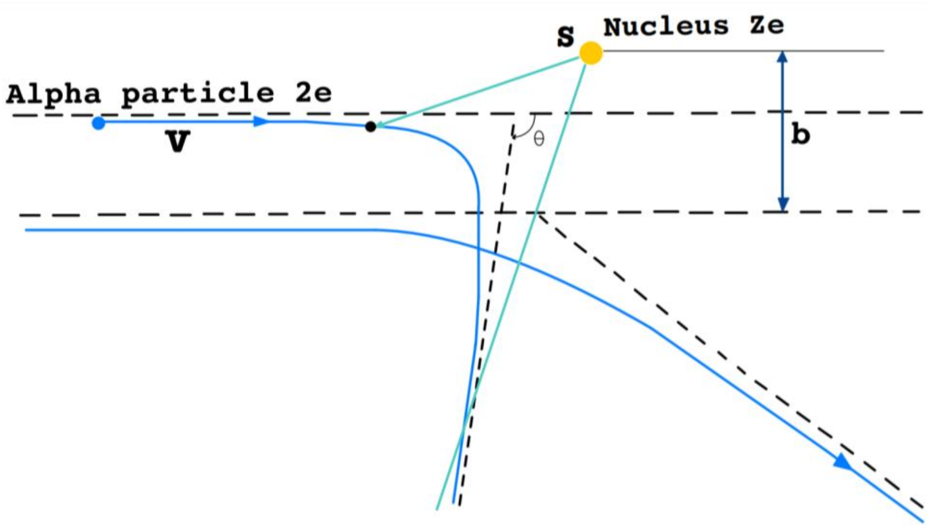
Then, the source provides alpha particles. These particles are accelerated to high energy and bombard the gold foil, and they are detected at different angles by the fluorescent screen. To observe the reflected particles, they used the scintillation method that allowed to understand where the particles hit the screen and, so, their angles of scattering.

They wondered what can stop an alpha particle and turn it back in its track. They agreed that it must be something with a mass.

In addition, they asked themselves why only few particles (about 1 in 10000) turned back with a great deflection. The most accredited answer was that, since the alpha particle and what it hit were very small, in most cases they went straight because they did not run across the (little) mass.

Then, they figured out that the responsible force was the electrical repulsion between the positively charged alpha particle and the central part of the gold atoms.

Rutherford verified that Thomson's model couldn't meet the experimental results. In fact, the electron has a mass that is too small, and a negative charge: the attraction between it and the particle would transmit the electron an acceleration, but it's impossible because the electron doesn't change its position and transmit the α-particle a negligible deviation.



- *b* is the distance between the initial direction of the **α**-particle and the nucleus, it is known as "cross parameter".

- The gold nucleus is *S*:
  - *Ze* is the gold nucleus charge
  - *M* is the nucleus mass
- Alpha particle charge:
  - *2e* is the alpha particle charge
  - *m* is the alpha particle mass
  - *v* is the initial speed of the particle
  - *s* is the initial direction of the alpha particle
- *r* is the distance between the particle and the nucleus

In Rutherford's model, Coulomb force acts on the alpha particle:

$$F = k\frac{2Ze^2}{r^2}$$

So, the closer the **α**-particle is to the nucleus, the stronger the repulsion force is and the greater the deflection is.

It has been demonstrated that alpha particles follow hyperbolic trajectories. The eccentricity of the hyperbola ($e = \frac{c}{a}$, this value indicates how much the hyperbola is flattened) increases depending on the value of *b*.

In the borderline case in which the trajectory crosses the nucleus, the particle reaches the minimum distance, then it goes back following the same path ($\boldsymbol{e} = \mathbf{1}$, for values close to 1 the hyperbola flattens on the x axis).

θ is the deflection angle. It is defined by the direction the particle assumes after the impact and the initial direction.

Cross section **σ** (sigma): it is a measure of probability that an alpha-particle will be deflected by a given angle during a collision with the atomic gold nucleus. It is expressed in units of transverse area. It can be figured as the area around the alpha particle which is effective for the scattering process.

$$\sigma = b \cdot \pi^2$$

b is the impact parameter. It is the centre-to-centre offset distance between the incident particle and the target, so that it is possible to draw a straight line for the incident path of the alpha particles and then how far it is from a parallel axis that runs through the centre of the target.

The relation between *b* and the deflection angle θ is defined by the formula:

$$b = \frac{k}{tan\frac{\theta}{2}}$$

If $\theta \to 0$, $b = \frac{2k}{\theta}$

K is a constant related to the product of the incident particles charges (the alpha particle with charge +2e and the nucleus with charge +Ze), the alpha particle mass and its starting speed.

$$k = \frac{2Ze^2}{mv^2}$$

So, *b* and θ are inversely proportional. In fact, the further from the nucleus the particle is (so the greater *b* is), the littler the angle is.

When $b \to \infty$, so when *b* is much bigger than the nucleus radius, $\theta \to 0$. It means that the particle is not deflected and it travels in a straight line.

It is easier to measure the intensity of the scattering at a given angle than the total number of particles. In fact, Rutherford deduces a formula known as "Rutherford formula" that gives the number of α-particles scattered by an angle θ.

$$n = \frac{C}{\sin^4\frac{\theta}{2}}$$

C is a constant value.

$$-\pi \leq \theta \leq +\pi$$

For $\theta \to 0$, $\lim_{\theta\to 0} n = \lim_{\theta \to 0} \frac{C}{\sin^4\frac{\theta}{2}} = \infty$.

This result is not acceptable in Physics, because it would mean that $b \to \infty$. The maximum distance from the particle and the nucleus should correspond to the radius of the gold atom ($\sim 1{,}5 \cdot 10^{-15} m$), which is not an infinite value, actually.

If $\boldsymbol{\alpha}$-particles speed is supposed to be $\frac{1}{100}c \sim 3 \cdot 10^6 \frac{m}{s}$, the minimum value of θ is:

$$\theta = \frac{2k}{b} \cong 10^{-14} m$$

It is a very low value, but it refuses the singularity.

In conclusion, the tendency of the number of particles as a function of the angle in a neighbourhood of $[-\pi, +\pi]$ is:

$$n = \frac{1}{\sin^4 \frac{\theta}{2}}$$

Rutherford demonstrated that the probability that a particle is scattered by an angle θ is a function related to $\frac{1}{\sin^4 \frac{\theta}{2}}$. The number of particles that aren't scattered increases by moving away from the nucleus. The particles scattered by a big angle are much less because the nucleus of the gold atom is very small in comparison to the entire atom. When the angle is equal to 180° the particle turns back, in fact at the value "π" the function reaches its minimum. From the mathematic point of view, at the value "$x = 0$" there is a vertical asymptote, it means that *b* can reach infinite values. In Physics the maximum value of *b* isn't infinite, but it corresponds to the radius of the atom.

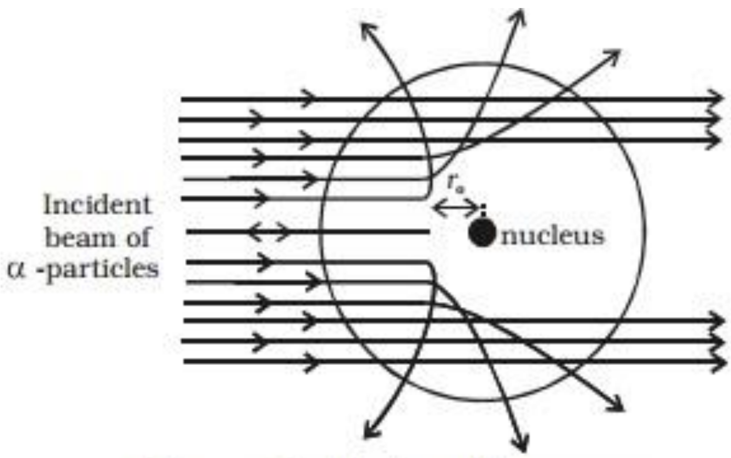
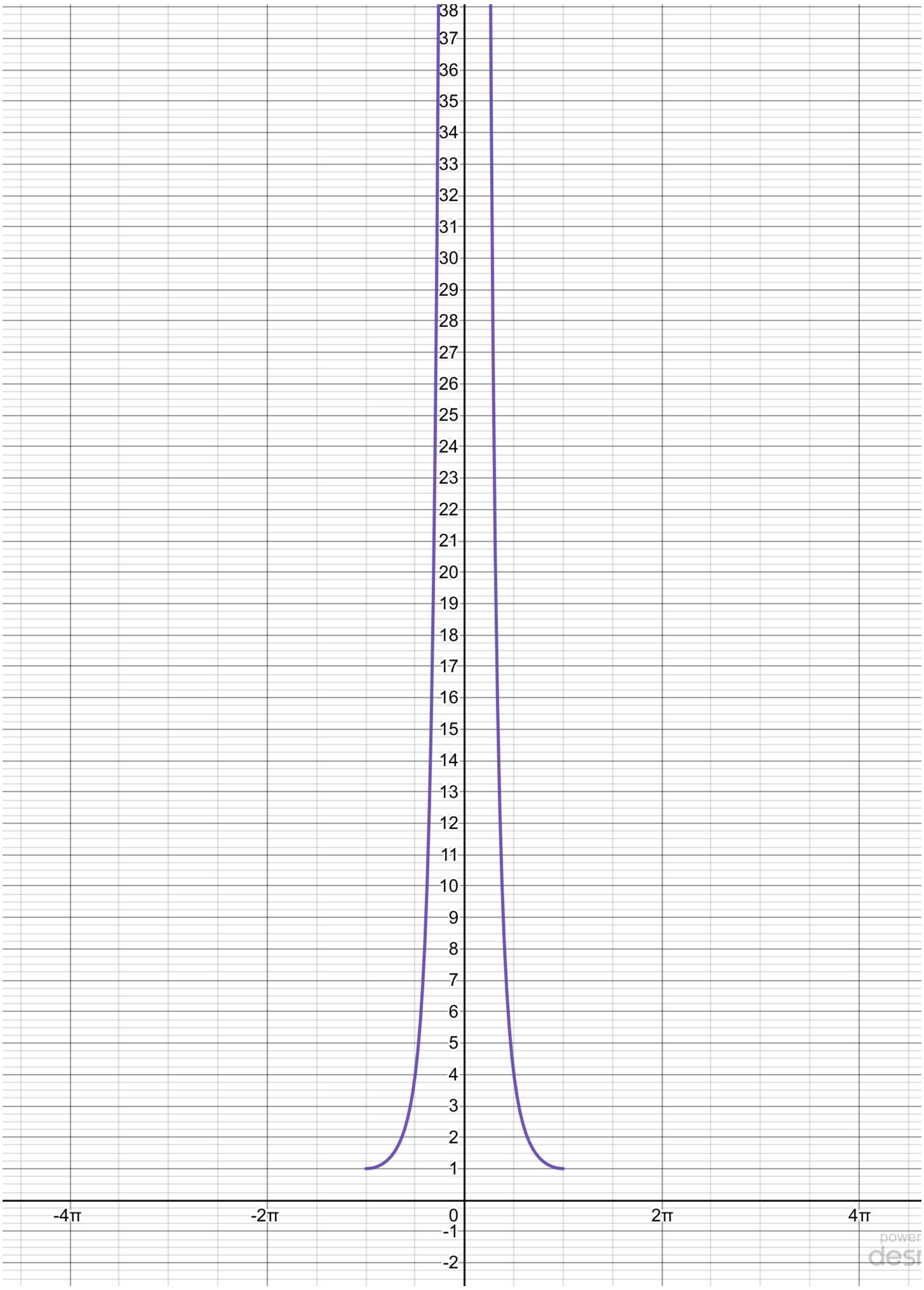


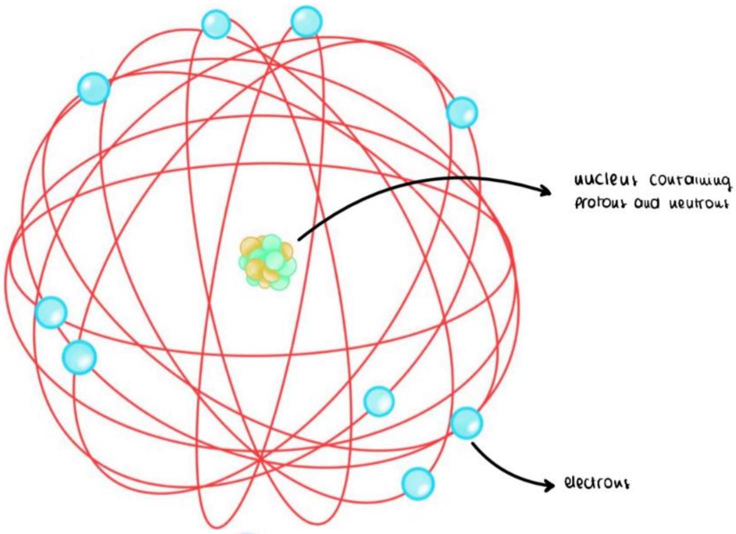
Fig Scattering of α – rays



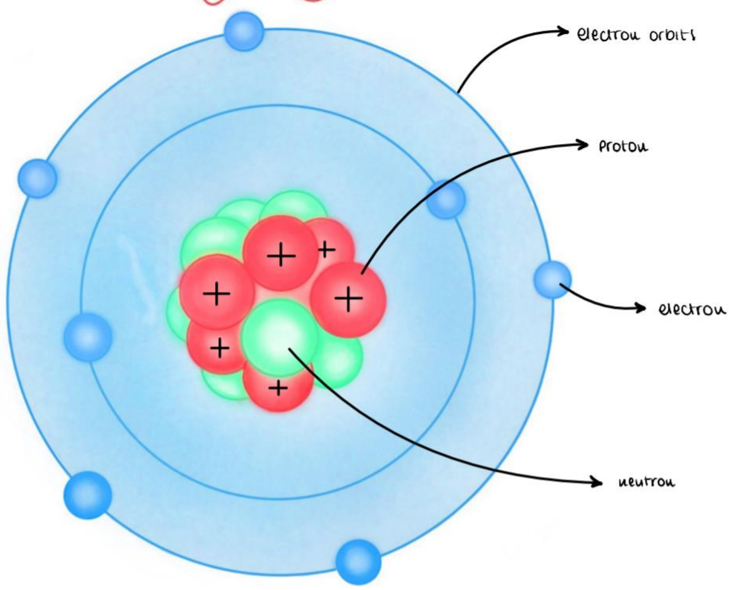
Experimental results

Even if the atomic model they proposed was soon defeated by Bohr's one, this experiment led to an important conclusion: atoms consist of a centralised positive mass that is very small in relation to the entire atom structure. The evidence suggested that an atom is mostly empty space and that its positive charge is concentrated in a dense mass at its core, forming a nucleus. When the positively charged alpha particles are fired at the gold foil, most of them passed through the empty space of the gold atoms with little deflection, but a few of them ran into the nucleus of a gold atom and were repelled straight back.

Rutherford proposed the planetary atomic model. According to it, the atom is composed by a tiny, dense, positive nucleus in which the mass is concentrated, while the electrons circulate around it as planets revolving around the Sun. Rutherford's model doesn't differentiate between any of the electrons, while Bohr's one places electrons into orbits with set energy levels.



Rutherford's model



Bohr's model

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Presentazione e materiali ricevuti dalla docente F. De Bernardi